DOI:
10.1364/OL.43.000963

Document Version
Early version, also known as pre-print

Electric current induced unidirectional propagation of surface plasmon-polaritons

K. Y. BLIOKH[1,2,*], F. J. RODRÍGUEZ-FORTUÑO[3], A. Y. BEKSHAEV[4], Y. S. KIVSHAR[2], AND F. NORI[1,5]

[1] Center for Emergent Matter Science, RIKEN, Wako-shi, Saitama 351-0198, Japan
[2] Nonlinear Physics Centre, The Australian National University, Canberra 0200, ACT, Australia
[3] Department of Physics, King's College London, London WC2R 2LC, UK
[4] I. I. Mechnikov National University, Dvorianska 2, Odessa, 65082, Ukraine
[5] Physics Department, University of Michigan, Ann Arbor, Michigan 48109-1040, USA
[*] Corresponding author: kostiantyn.bliokh@riken.jp

Compiled December 1, 2017

Nonreciprocity and one-way propagation of optical signals is crucial for modern nanophotonic technology, and is typically achieved using magneto-optical effects requiring large magnetic biases. Here we suggest a fundamentally novel approach to achieve unidirectional propagation of surface plasmon-polaritons (SPPs) at metal-dielectric interfaces. We employ a direct electric current in metals, which produces the Doppler frequency shift of SPPs due to the uniform drift of electrons. This tilts the dispersion of SPPs, enabling one-way propagation, as well as zero and negative group velocities. The results are demonstrated for planar interfaces and cylindrical nanowire waveguides.

OCIS codes: (240.6680) Surface plasmons; (250.5403) Plasmonics; (230.3810) Magneto-optic systems

http://dx.doi.org/10.1364/ao.XX.XXXXXX

Nonreciprocity and unidirectional propagation of electromagnetic waves are highly important topics in modern optics, crucial for nanophotonic, quantum-optical, and optoelectronic applications [1–10]. The main mechanisms generating one-way propagation and strong nonreciprocity are: magnetooptical phenomena [2, 3, 9–11], including topological quantum-Hall effect [2, 3], nonlinearity resulting in optical diodes and circulators [4, 8, 12–14], and other methods breaking the time-reversal symmetry in the system [5, 6].

The study of surface waves and plasmonics is another inherent part of nanophotonics, which allows to reduce the scales and dimensionality of various electromagnetic phenomena [15, 16]. Not surprisingly, nonreciprocity and unidirectional propagation of surface plasmon-polaritons (SPPs) have recently attracted considerable attention [17–22]. These studies mostly deal with magnetooptical nonreciprocity in the transverse Voigt geometry, including topological quantum-Hall-effect states [20, 23, 24].

Here we put forward a novel mechanism resulting in one-way propagation of SPPs at metal-dielectric interfaces. Namely, we show that in the presence of a longitudinal direct electric current, the SPP spectrum becomes nonreciprocal, with a unidirectional propagation in a certain frequency range. This is caused by the Doppler shift of the wave frequency in the drifting electron plasma. Furthermore, the SPP spectrum is deformed such that the group velocity of SPPs propagating along the current vanishes at a critical wavevector, and then becomes negative for larger wavevectors. Thus, the electric-current-induced nonreciprocity is qualitatively different as compared with the known magnetic field-induced case.

Importantly, we show that the nonreciprocal effect from the electric current can be comparable with the magnetooptical one at reasonable values of the system parameters. Moreover, we consider SPPs at a planar metal-dielectric interface, as well as in a cylindrical nanowire. Metallic nanowires provide a highly efficient platform for plasmonics and metamaterials [25–28], and they can be naturally biased by a direct electric current. As we show below, this results in the nonreciprocal properties of nanowire plasmons.

To start with, we consider SPPs propagating along the planar metal-vacuum interface $x = 0$, in the $\pm z$ directions, as shown in Fig. 1. We employ the simplest Drude model of the metal (neglecting losses) with the permittivity $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$ and plasma frequency ω_p. It is well known [15, 16] that SPPs exist at frequencies $\omega < \omega_p/\sqrt{2}$, i.e., $\varepsilon < -1$, and propagate along the interface with the wavevector $\mathbf{k}_p = k_p\,\bar{\mathbf{z}}$, $k_p = \sigma k_0\sqrt{-\varepsilon}/\sqrt{-1-\varepsilon}$, $|k_p| > k_0$. Hereafter, the overbar denotes the unit vectors of the corresponding axes, $k_0 \equiv \omega/c$, and we introduced the parameter $\sigma = \operatorname{sgn} k_p = \pm 1$ indicating the SPP propagation direction. The dependence $k_p(\omega)$ determines the dispersion relation of SPPs (see the dashed curve in Fig. 2). The SPP field decays away from the interface with the exponential-decay rates $\kappa_1 = k_p/\sqrt{-\varepsilon}$ (in the vacuum, $x > 0$) and $\kappa_2 = \sqrt{-\varepsilon}\,k_p$ (in the metal, $x < 0$).

We first briefly describe the nonreciprocity and unidirectional propagation of SPPs in the presence of a transverse magnetic field $\boldsymbol{\mathcal{H}} = \mathcal{H}\,\bar{\mathbf{y}}$ [18–20]. Usually, it is calculated using the anisotropic permittivity tensor of the magnetoactive metal. However, we employ a simpler way to derive the same results. Recently, some of us have shown [29, 30] that the (x, z)-plane

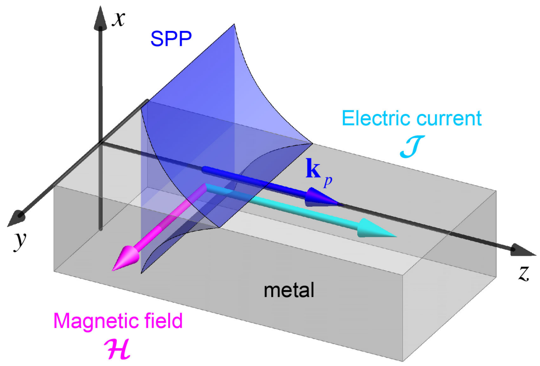

Fig. 1. Schematic picture of a surface plasmon-polariton (SPP) propagating along the metal-vacuum interface [15, 16]. The applied magnetic field $\boldsymbol{\mathcal{H}}$, and direct electric current $\boldsymbol{\mathcal{J}}$ are shown.

rotation of the electric field of the SPP induces the corresponding orbital motion of electrons in the metal and, hence, the *transverse magnetization* due to the inverse Faraday effect. (This property is related to the transverse spin of SPPs [31], which is currently attracting considerable attention [32–34].) Using Gaussian units, the magnetization of the metal can be written as [29, 30]

$$\mathbf{M} = \frac{ge\omega}{4mc}\frac{d\varepsilon}{d\omega}\,\mathrm{Im}(\mathbf{E}^* \times \mathbf{E}) = \sigma\, g\, \frac{|E_0|^2}{\sqrt{-\varepsilon}}\frac{-e}{mc}\exp(2\kappa_2 x)\,\bar{\mathbf{y}}. \qquad (1)$$

Here, $g = (8\pi\omega)^{-1}$, $\mathbf{E}\,(\mathbf{r})$ is the complex electric field in the SPP wave, omitting $\exp(-i\omega t)$, E_0 is its amplitude right above the metal, whereas $e < 0$ and m are the electron charge and mass, respectively. The magnetization (1) means that SPPs, being mixed light-electron quasiparticles, carry *transverse magnetic moment* $\boldsymbol{\mu} \propto \sigma\,\bar{\mathbf{y}}$. It can be calculated as a ratio of the integral magnetization (1) to the number of the quasiparticles. Using the standard Brillouin energy density W, this yields [29, 30]:

$$\boldsymbol{\mu} = \frac{\hbar\omega}{\langle W\rangle}\,\langle \mathbf{M}\rangle = \sigma\,\frac{2\sqrt{-\varepsilon}}{1+\varepsilon^2}\,\mu_B\,\bar{\mathbf{y}}, \qquad (2)$$

where $\langle ...\rangle = \int ...\, dx$, and $\mu_B = \hbar\,|e|\,/2mc$ is the Bohr magneton. The absolute value of the magnetic moment (2) grows from 0 to μ_B as the SPP frequency ω changes from 0 to $\omega_p/\sqrt{2}$.

Equations (1) and (2) describe intrinsic properties of SPPs *without* an external magnetic field. Applying the magnetic field $\boldsymbol{\mathcal{H}} = \mathcal{H}\,\bar{\mathbf{y}}$ leads to the *Zeeman interaction* with the magnetic moment (2), $-\boldsymbol{\mu}\cdot\boldsymbol{\mathcal{H}}$, which shifts the energy (frequency) of the SPP [35]. Denoting the SPP frequency without magnetic field as $\omega_0\,(k_p)$, the Zeeman-shifted frequency in an external magnetic field becomes $\omega(k_p) = \omega_0(k_p) + \delta\omega(k_p)$:

$$\delta\omega = -\hbar^{-1}\boldsymbol{\mu}\cdot\boldsymbol{\mathcal{H}} = -\frac{\sqrt{-\varepsilon}}{1+\varepsilon^2}\,\sigma\,\Omega. \qquad (3)$$

Here, $\Omega = -e\mathcal{H}/mc$ is the cyclotron frequency of the electrons in the magnetic field $\mathcal{H}$, and the correction $\delta\omega$ depends on k_p via $\varepsilon\left[\omega_0(k_p)\right]$. The modified SPP dispersion (3) is shown in Fig. 2(a). The magnetic correction makes the spectrum *nonreciprocal*, i.e., depending on the propagation direction σ. In particularly, the cutoff frequency $\omega_p/\sqrt{2}$ is now shifted to $\omega_p/\sqrt{2} + \sigma\,\Omega/2$. This means that in the range $\omega \in \left(\omega_p/\sqrt{2} - \Omega/2, \omega_p/\sqrt{2} + \Omega/2\right)$, SPPs become *unidirectional*, i.e., propagating only in the positive

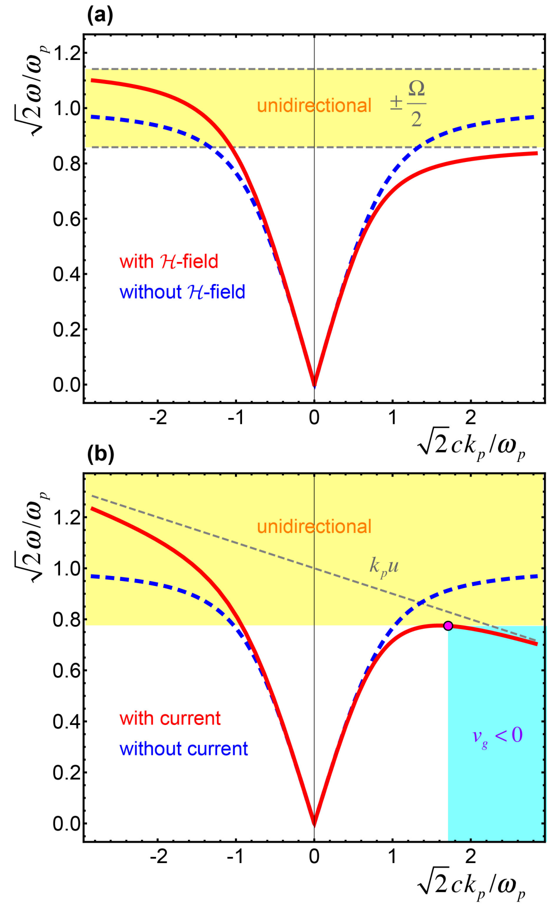

Fig. 2. Nonreciprocal modifications of the SPP spectra caused by a transverse magnetic field (a), Eq. (3), and a longitudinal direct electric current (b), Eqs. (5) and (6). The dashed curves show the unperturbed reciprocal SPP dispersion $\omega_0\,(k_p)$. The frequency ranges with the one-way SPP propagation and the wavevector range with the negative group velocity of SPPs are marked by yellow and blue, respectively. The parameters are $\Omega = 0.2\,\omega_p$ (a) and $u = -0.1\,c$ (b).

(negative) z-direction for $\mathcal{H} > 0$ ($\mathcal{H} < 0$). Notably, the magnetic correction to the dispersion (3) exactly coincides with the one calculated in [18] using anisotropic permittivity of the metal in a magnetic field.

We are now in the position to consider SPPs in the presence of a direct electric current with the density $\boldsymbol{\mathcal{J}} = \mathcal{J}\,\bar{\mathbf{z}}$ flowing in the metal. In this case, the problem can be readily analyzed in terms of the modified permittivity $\varepsilon\,(\omega)$. Indeed, the presence of the current means that free electrons in the metal move with the velocity $\mathbf{u} = \boldsymbol{\mathcal{J}}/ne$, where $n = m\omega_p^2/4\pi e^2$ is the volume density of the electrons. This movement of the electron plasma produces the *Doppler frequency shift* $\omega \to \omega - k\,u$ in the metal permittivity [36]:

$$\varepsilon\,(\omega) = 1 - \frac{\omega_p^2}{(\omega - k_p u)^2}. \qquad (4)$$

Considering the z-aligned propagation of SPPs, we can still use the usual form of the SPP dispersion relation, $k_p = k_0\,\sqrt{-\varepsilon}/\sqrt{-1-\varepsilon}$, but now with the Doppler-modified permittivity (4). Expanding this in the linear approximation in the drift velocity u, we arrive at the following dispersion relation:

$$\omega = \omega_0(k_p) + \frac{1-\varepsilon}{1+\varepsilon^2}\,\sigma\,|k_p|\,u. \qquad \textbf{(5)}$$

The current-modified SPP dispersion (5) is nonreciprocal, as shown in Fig. 2(b). Moreover, this nonreciprocity differs qualitatively from the known magnetic-field case, Fig. 2(a). Indeed, the cut-off frequency asymptote $\omega_p/\sqrt{2}$ (for $|k_p| \to \infty$) is now *tilted* as $\omega_p/\sqrt{2} + k_p u$, rather than split. The most interesting feature of the modified dispersion is that it has an *inflexion point*:

$$k_p^{\rm inf} = -\frac{\omega_p}{\sqrt{2}\,c}\left(\frac{c}{2\,u}\right)^{\frac{1}{3}}, \quad \omega\left(k_p^{\rm inf}\right) = \frac{\omega_p}{\sqrt{2}} + \frac{3}{2}\,k_p^{\rm inf}\,u. \qquad \textbf{(6)}$$

The SPP group velocity $v_g = \partial\omega/\partial k_p$ vanishes and changes its sign in this point. For positive current $\mathcal{J} > 0$, $u < 0$, $k_p^{\rm inf} > 0$, and the group velocity becomes negative for $k > k_p^{\rm inf}$. This is because slow SPPs near the cut-off frequency $\omega_p/\sqrt{2}$ are drifted by the flow of electrons in the backward direction. Furthermore, the inflexion point (6) determines the maximum frequency of the SPPs propagating along the current $\mathcal{J}$. For $\omega > \omega\left(k_p^{\rm inf}\right)$, SPPs become *unidirectional*, propagating only in the direction opposite to the current. Due to the tilt of the cut-off asymptote, the unidirectional-propagation range is not limited from above by a higher frequency. However, practically, high wave numbers $|k_p|$ are accompanied by strong absorption of SPPs [16].

The inflexion-point parameters are determined by the ratio of the electron drift velocity to the speed of light: $|u|/c \ll 1$. For typical laboratory currents, this is a very small parameter. However, the power of 1/3 makes the inflexion-point characteristics not too extreme, resulting in observable consequences at feasible parameters. In particular, the current-induced cut-off frequency shift $\sim k_p^{\rm inf} u$ could be of the order of or even larger than the similar magnetic-field-induced shift $\sim \Omega$.

For example, the work [21] considered a gold nanowire of ~~the~~ radius $r_0 = 10^{-5}$ cm in the presence of an electric current $\mathcal{I} = \pi r_0^2 \mathcal{J} = 75 \cdot 10^{-3}$ A, see Fig. 3. Using the free-electron density in gold, $n \simeq 6 \cdot 10^{22}\,\text{cm}^{-3}$, we find the electron drift velocity $|u| \simeq 2.5 \cdot 10^4$ cm/s $\simeq 0.8 \cdot 10^{-6}\,c$. The SPP cut-off frequency was $\omega_p/\sqrt{2} \simeq 4.8 \cdot 10^{15}\,\text{s}^{-1}$, and the SPP wave number $|k_p| \simeq 2 \cdot 10^7\,\text{cm}^{-1}$. The work [21] examined the nonreciprocal effect of the azimuthal magnetic field generated by the current, $\mathcal{H}_\varphi = 2\mathcal{I}/cr_0 \simeq 1.5 \cdot 10^3$ G (and enhanced by a magnetoactive dielectric around the wire), but neglected the direct electric-current effect on surface plasmons. In fact, the above parameters correspond to the cyclotron frequency $\Omega \simeq 3 \cdot 10^{10}\,\text{s}^{-1}$ and the Doppler frequency shift $|k_p u| \simeq 5 \cdot 10^{11}\,\text{s}^{-1} \gg \Omega$. Thus, the electric-current nonreciprocity is one order of magnitude stronger than the magnetic-field nonreciprocity (in pure metal, without a magnetoactive dielectric) for these parameters.

To properly analyze the electric-current effect in a nanowire, we now consider SPPs in the cylindrical geometry of a metallic wire of radius r_0, Fig. 3. For the sake of generality, we introduce permittivity $\varepsilon_2 < 0$ and permeability $\mu_2 > 0$ inside the wire, and permittivity $\varepsilon_1 > 0$ and permeability $\mu_1 > 0$ outside the wire (later we set $\varepsilon_1 = \mu_1 = \mu_2 = 1$). The analytical form of plasmonic wire modes can be obtained as follows. Since these modes are TM polarized, they can be described entirely by the vector potentials $\mathbf{A} = A\,\bar{\mathbf{z}}$ and $\mathbf{F} = 0$ [37], where A is a general solution of the scalar wave equation in cylindrical coordinates

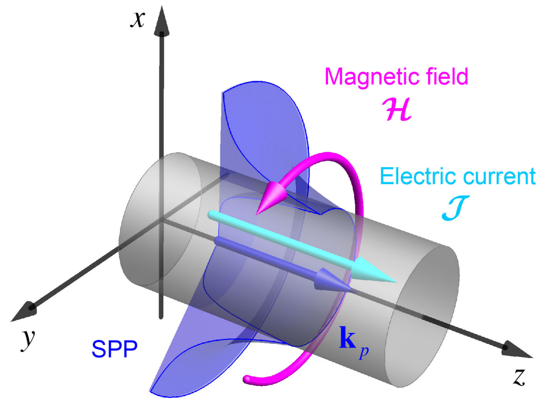

Fig. 3. Schematic picture of an SPP mode of a metallic nanowire. The direct electric current $\mathcal{J}$ in the nanowire and the corresponding induced magnetic field $\mathcal{H}$ (considered in [21]) are shown.

(r, φ, z):

$$A = A_0\, e^{i\ell\varphi + ik_p z} \begin{cases} a\, I_\ell(\kappa_2 r)\,, & r < r_0\,, \\ b\, K_\ell(\kappa_1 r)\,, & r > r_0\,. \end{cases} \qquad \textbf{(7)}$$

Here, $\ell = 0, \pm1, \pm2, \ldots$ is the azimuthal (angular-momentum) quantum number, k_p is the propagation constant, $\kappa_{1,2} = \sqrt{k_p^2 - k_{1,2}^2}$ are the radial exponential-decay constants, $k_{1,2} = \sqrt{\varepsilon_{1,2}\mu_{1,2}}\,k_0$ are the wave numbers in the two media, while I_ℓ and K_ℓ are the modified Bessel functions. The amplitudes (a, b) in Eq. (7) are to be determined. The wave electric and magnetic fields in each medium can be derived from the vector potentials through the relations [37]:

$$\mathbf{E} = ik_0\mathbf{A} + \frac{ik_0}{k_{1,2}^2}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{A}) - \frac{1}{\varepsilon_{1,2}}\boldsymbol{\nabla}\times\mathbf{F}\,,$$
$$\mathbf{H} = ik_0\mathbf{F} + \frac{ik_0}{k_{1,2}^2}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{F}) + \frac{1}{\mu_{1,2}}\boldsymbol{\nabla}\times\mathbf{A}\,. \qquad \textbf{(8)}$$

Substituting the potential (7) into Eqs. (8), we obtain all vector components of the wave fields. From now on, we consider the fundamental mode with $\ell = 0$. Applying the electromagnetic boundary conditions at $r = r_0$, we arrive at the system of equations for the amplitudes (a, b):

$$\hat{M}\begin{pmatrix} a \\ b \end{pmatrix} \equiv \begin{pmatrix} \kappa_2^2\varepsilon_1\mu_1 I_0(\rho_2) & -\kappa_1^2\varepsilon_2\mu_2 K_0(\rho_1) \\ 2\kappa_2\mu_1 I_1(\rho_2) & 2\kappa_1\mu_2 K_1(\rho_1) \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0, \qquad \textbf{(9)}$$

where $\rho_{1,2} = \kappa_{1,2} r_0$. Equation (9) has non-trivial solutions only when $D \equiv \det\hat{M} = 0$, which provides the transcendental characteristic equation $D(\omega, k_p) = 0$ for the plasmonic mode dispersion.

Similarly to the planar SPP case, we introduce the effect of the electric current via the Doppler shift (4) in the Drude-metal permittivity $\varepsilon_2 \equiv \varepsilon(\omega)$. The drift velocity of the electrons is related to the current as $\mathcal{I} = \pi r_0^2 \mathcal{J} = \pi r_0^2\, neu$. Substituting the Doppler-modified permittivity (4) into the characteristic equation, we numerically find the modified dispersion relation for the fundamental SPP mode in the electric-biased nanowire. Figure 4 shows the dispersion relation for a nanowire with $\omega_p = 10^{16}$ rad/s, $r_0 = 20$ nm, and different values of $\mathcal{I}$.

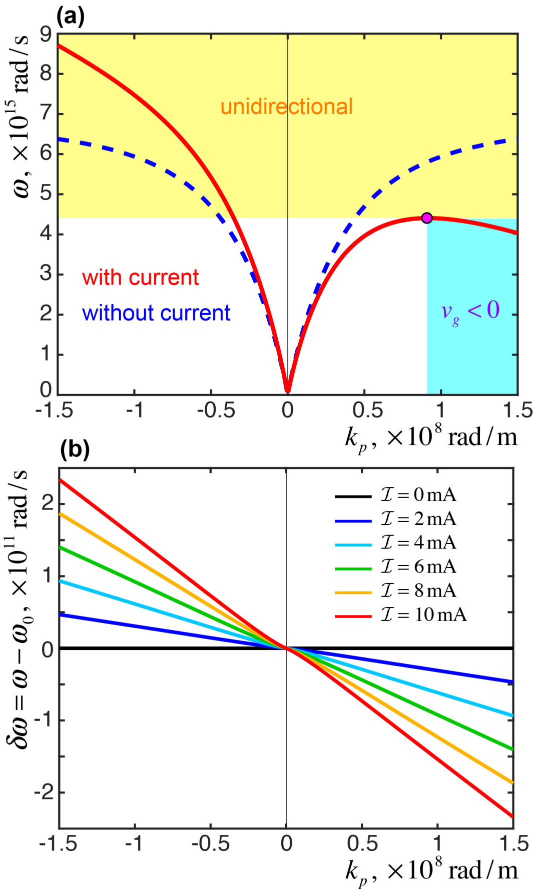

Fig. 4. Nonreciprocal electric current-induced modifications of the SPP spectra in a metallic nanowire with $\omega_p = 10^{16}$ rad/s and radius $r_0 = 20$ nm. (a) The unidirectional-propagation and negative-group-velocity ranges are shown for a very high current $\mathcal{I} = 100$ A, cf. Fig. 2b. (b) Small nonreciprocity from realistic currents $\mathcal{I} \leq 10$ mA is depicted in the form of the deflection $\delta\omega(k_p)$ from the reciprocal dispersion $\omega_0(k_p)$.

Panel (a) shows the modified dispersion for a very high value of the current $\mathcal{I} = 100$ A to exaggerate the nonreciprocal effect, while panel (b) displays the zoomed-in perturbation of the SPP dispersion for realistic smaller currents $\mathcal{I} \leq 10$ mA. All the features discussed for the planar SPP, including the one-way propagation and negative group velocity ranges, can be clearly observed here.

In conclusion, we have proposed a simple but yet fundamental way to achieve unidirectional propagation of surface plasmon-polaritons using a direct electric current in metals. The one-way propagation of optical signals, in analogy to electronic isolators, is considered as a fundamental requirement for enabling photonic high-speed all-optical processing that could substitute current microelectronic components. Nonreciprocal propagation requires breaking the time-reversal symmetry in the system. This is usually done via magneto-optical effects requiring large magnetic biases. In contrast, our proposal is based on the use of an electric current, which can be naturally generated in plasmonic waveguides. The ability to achieve one-way optical propagation using direct electric currents is conceptually simple and inherently compatible with modern microelectronics industry. This opens up new avenues towards all-optical miniaturized processing components.

We acknowledge helpful discussions with Y. P. Bliokh, N. Noginova, and M. Durach. This work was supported by the RIKEN iTHES Project, MURI Center for Dynamic Magneto-Optics via the AFOSR Award No. FA9550-14-1-0040, the Japan Society for the Promotion of Science (KAKENHI), the IMPACT program of JST, CREST grant No. JPMJCR1676, the John Templeton Foundation, European Research Council (Starting Grant ERC-2016-STG-714151-PSINFONI), and the Australian Research Council.

REFERENCES

1. R. J. Potton, Rep. Prog. Phys. **67**, 717 (2004).
2. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljačić, Phys. Rev. Lett. **100**, 013905 (2008).
3. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljačić, Nature **461**, 772 (2009).
4. L. Fan *et al.*, Science **335**, 447 (2012).
5. H. Lira, Z. Yu, S. Fan, and M. Lipson, Phys. Rev. Lett. **109**, 033901 (2012).
6. D. L. Sounas, C. Caloz, and A. Alu, Nature Commun. **4**, 2407 (2013).
7. A. B. Khanikaev *et al.*, Nature Mater. **12**, 233 (2013).
8. B. Peng *et al.*, Nature Phys. **10**, 394 (2014).
9. Y. Hadad and B. Z. Steinberg, Phys. Rev. Lett. **105**, 233904 (2010).
10. C. Sayrin *et al.*, Phys. Rev. X **5**, 041036 (2015).
11. A. K. Zvezdin and V. A. Kotov, *Modern magnetooptics and magnetooptical materials* (IoP publishing, 1997).
12. M. Scalora, J. P. Dowling, C. M. Bowden, and M. J. Bloemer, J. Appl. Phys. **76**, 2023 (1994).
13. M. D. Tocci, M. J. Bloemer, M. Scalora, J. P. Dowling, and C. M. Bowden, Appl. Phys. Lett. **66**, 2324 (1995).
14. M. W. Feise, I. V. Shadrivov, and Y. S. Kivshar, Phys. Rev. E **71**, 037602 (2005).
15. A. V. Zayats, I. I. Smolyaninov, and A. A. Maradudin, Phys. Rep. **408**, 131 (2005).
16. S. A. Maier, *Plasmonics: Fundamentals and Applications* (Springer, 2007).
17. R. E. Camley, Surface Sci. Rep. **7**, 103 (1987).
18. Z. Yu, G. Veronis, Z. Wang, and S. Fan, Phys. Rev. Lett. **100**, 023902 (2008).
19. B. Hu, Q. J. Wang, and Y. Zhang, Opt. Lett. **37**, 1895 (2012).
20. A. R. Davoyan and N. Engheta, Phys. Rev. Lett. **111**, 257401 (2013).
21. A. R. Davoyan and N. Engheta, Nature Commun. **5**, 5250 (2014).
22. I. S. Maksymov, Nanomaterials **5**, 577 (2015).
23. M. G. Silveirinha, Phys. Rev. B **92**, 125153 (2015).
24. M. G. Silveirinha, Phys. Rev. B **93**, 075110 (2016).
25. H. Ditlbacher *et al.*, Phys. Rev. Lett. **95**, 257403 (2005).
26. M. I. Stockman, Phys. Rev. Lett. **93**, 137404 (2004).
27. A. V. Akimov *et al.*, Nature **450**, 402 (2007).
28. C. R. Simovski, P. A. Belov, A. V. Atrashchenko, and Y. S. Kivshar, Adv. Mater. **24**, 4229 (2012).
29. K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, Phys. Rev. Lett. **119**, 073901 (2017).
30. K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, New J. Phys. (2017) https://doi.org/10.1088/1367-2630/aa8913.
31. K. Y. Bliokh and F. Nori, Phys. Rev. A **85**, 061801(R) (2012).
32. K. Y. Bliokh and F. Nori, Phys. Rep. **592**, 1 (2015).
33. A. Aiello, P. Banzer, M. Neugebauer, and G. Leuchs, Nature Photon. **9**, 789 (2015).
34. K. Y. Bliokh, F. J. Rodríguez-Fortuño, F. Nori, and A. Zayats, Nature Photon. **9**, 796 (2015).
35. L. Landau and E. M. Lifshitz, *Quantum Mechanics* (Butterworth-Heinemann, 1977).
36. E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Butterworth-Heinemann, 1981).
37. C. A. Balanis, *Advanced Engineering Electromagnetics* (Wiley, 1989).

REFERENCES

1. R. J. Potton, “Reciprocity in optics”, Rep. Prog. Phys. **67**, 717–754 (2004).
2. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljačić, “Reflection-free one-way edge modes in a gyromagnetic photonic crystal”, Phys. Rev. Lett. **100**, 013905 (2008).
3. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljačić, “Observation of unidirectional backscattering-immune topological electromagnetic states”, Nature **461**, 772–775 (2009).
4. L. Fan *et al.*, “An all-silicon passive optical diode”, Science **335**, 447–450 (2012).
5. H. Lira, Z. Yu, S. Fan, and M. Lipson, “Electrically driven nonreciprocity induced by interband photonic transition on a silicon chip”, Phys. Rev. Lett. **109**, 033901 (2012).
6. D. L. Sounas, C. Caloz, and A. Alu, “Giant non-reciprocity at the subwavelength scale using angular momentum-biased metamaterials”, Nature Commun. **4**, 2407 (2013).
7. A. B. Khanikaev *et al.*, “Photonic topological insulators”, Nature Mater. **12**, 233–239 (2013).
8. B. Peng *et al.*, “Parity-time-symmetric whispering-gallery microcavities”, Nature Phys. **10**, 394–398 (2014).
9. Y. Hadad and B. Z. Steinberg, “Magnetized spiral chains of plasmonic ellipsoids for one-way optical waveguides”, Phys. Rev. Lett. **105**, 233904 (2010).
10. C. Sayrin *et al.*, “Nanophotonic optical isolator controlled by the internal state of cold atoms”, Phys. Rev. X **5**, 041036 (2015).
11. A. K. Zvezdin and V. A. Kotov, *Modern magnetooptics and magnetooptical materials* (IoP publishing, 1997).
12. M. Scalora, J. P. Dowling, C. M. Bowden, and M. J. Bloemer, “The photonic band edge optical diode”, J. Appl. Phys. **76**, 2023–2026 (1994).
13. M. D. Tocci, M. J. Bloemer, M. Scalora, J. P. Dowling, and C. M. Bowden, “Thin-film nonlinear optical diode”, Appl. Phys. Lett. **66**, 2324–2326 (1995).
14. M. W. Feise, I. V. Shadrivov, and Y. S. Kivshar, “Bistable diode action in left-handed periodic structures”, Phys. Rev. E **71**, 037602 (2005).
15. A. V. Zayats, I. I. Smolyaninov, and A. A. Maradudin, “Nano-optics of surface plasmon polaritons”, Phys. Rep. **408**, 131–314 (2005).
16. S. A. Maier, *Plasmonics: Fundamentals and Applications* (Springer, 2007).
17. R. E. Camley, “Nonreciprocal surface waves”, Surface Sci. Rep. **7**, 103–187 (1987).
18. Z. Yu, G. Veronis, Z. Wang, and S. Fan, “One-way electromagnetic waveguide formed at the interface between a plasmonic metal under a static magnetic field and a photonic crystal”, Phys. Rev. Lett. **100**, 023902 (2008).
19. B. Hu, Q. J. Wang, and Y. Zhang, “Broadly tunable one-way terahertz plasmonic waveguide based on nonreciprocal surface magneto plasmons”, Opt. Lett. **37**, 1895–1897 (2012).
20. A. R. Davoyan and N. Engheta, “Theory of wave propagation in magnetized near-zero-epsilon metamaterials: Evidence for one-way photonic states and magnetically switched transparency and opacity”, Phys. Rev. Lett. **111**, 257401 (2013).
21. A. R. Davoyan and N. Engheta, “Electrically controlled one-way photon flow in plasmonics nanostructures”, Nature Commun. **5**, 5250 (2014).
22. I. S. Maksymov, “Magneto-plasmonics and resonant interaction of light with dynamic magnetisation in metallic and all-magneto-dielectric nanostructures”, Nanomaterials **5**, 577–613 (2015).
23. M. G. Silveirinha, “Chern invariants for continuous media”, Phys. Rev. B **92**, 125153 (2015).
24. M. G. Silveirinha, “Z_2 topological index for continuous photonic materials”, Phys. Rev. B **93**, 075110 (2016).
25. H. Ditlbacher *et al.*, “Silver nanowires as surface plasmon resonators”, Phys. Rev. Lett. **95**, 257403 (2005).
26. M. I. Stockman, “Nanofocusing of optical energy in tapered plasmonic waveguides”, Phys. Rev. Lett. **93**, 137404 (2004).
27. A. V. Akimov *et al.*, “Generation of single optical plasmons in metallic nanowires coupled to quantum dots”, Nature **450**, 402–406 (2007).
28. C. R. Simovski, P. A. Belov, A. V. Atrashchenko, and Y. S. Kivshar, “Wire metamaterials: Physics and applications”, Adv. Mater. **24**, 4229–4248 (2012).
29. K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, “Optical momentum, spin, and angular momentum in dispersive media”, Phys. Rev. Lett. **119**, 073901 (2017).
30. K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, “Optical momentum and angular momentum in complex media: From the Abraham-Minkowski debate to unusual properties of surface plasmon-polaritons”, New J. Phys. (2017) https://doi.org/10.1088/1367-2630/aa8913.
31. K. Y. Bliokh and F. Nori, “Transverse spin of a surface polariton”, Phys. Rev. A **85**, 061801(R) (2012).
32. K. Y. Bliokh and F. Nori, “Transverse and longitudinal angular momenta of light”, Phys. Rep. **592**, 1–38 (2015).
33. A. Aiello, P. Banzer, M. Neugebauer, and G. Leuchs, “From transverse angular momentum to photonic wheels”, Nature Photon. **9**, 789–795 (2015).
34. K. Y. Bliokh, F. J. Rodríguez-Fortuño, F. Nori, and A. Zayats, “Spin-orbit interactions of light”, Nature Photon. **9**, 796–808 (2015).
35. L. Landau and E. M. Lifshitz, *Quantum Mechanics* (Butterworth-Heinemann, 1977).
36. E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Butterworth-Heinemann, 1981).
37. C. A. Balanis, *Advanced Engineering Electromagnetics* (Wiley, 1989).